

**THE 2021 2022 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION**

PART I MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing will lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. Consider the following 5 statements:

- (i) Statement (ii) is true.
- (ii) At most one of these 5 statements is true.
- (iii) All 5 of these statements are true.
- (iv)
- (v)

The last two statements are invisible. Which of the following is correct?

- (A) Only statement (i) is true. (B) Only statement (ii) is true.
- (C) Only one of statements (iv) or (v) is true. (D) Both statements (iv) and (v) are true.
- (E) Neither of statements (iv) and (v) are true.

2. Compute the sum of all values of $n \in \mathbb{Q} > 0$ such that the base 10 representation of $\frac{1}{n}$ is the square of an integer.

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

6. If $0 < \alpha < 90$, and $\sin \alpha = \frac{3}{5}$, what is the value of $\cos \alpha$?

19. Suppose L and M are positive real numbers for which $\sqrt[3]{L} = \sqrt[3]{M}$ and $\sqrt[3]{L} + \sqrt[3]{M} = 6$.

Compute the value of $\frac{L}{M}$.

(A) $\frac{\sqrt[3]{6}}{\sqrt[3]{9}}$

(B) $\frac{6}{\sqrt[3]{9}}$

(C) $\frac{\sqrt[3]{9} + 5}{8}$

(D) $\frac{\sqrt[3]{9} + 5}{6}$

(E) $\sqrt[3]{9} + 5$

9. **C** Let N be the number of red marbles and B be the number of blue marbles. Before drawing a marble, we know that the probability of drawing a blue marble is $\frac{B}{N+B} = \frac{5}{8}$. After drawing one marble, the probability of drawing a blue marble has decreased. Thus, a blue marble was drawn, and now the probability of drawing a blue marble is $\frac{B-1}{N+B-1} = \frac{5}{9}$. We can rewrite these two equations as $B = \frac{5}{8}(N+B)$ and $B-1 = \frac{5}{9}(N+B-1)$. Solving these two equations, we obtain $B = 4$ and $N = 12$, and hence there are 12 red marbles in the bag.
10. **D** We are given that $0 < L < T < E < T < E < s < E < T < E < t < L < u < T < E < u$ and also that $0 < L < U < E < U < E < s < E < U < E < t < E < U < E < u < L < v < U < E < x$ for some positive integers x and y . Therefore, $v < U < L < u < T < s$, which is satisfied for any value of y that is a multiple of 3. Since $v < U < E < x < Q < t < r < t < s < U < Q < w < r < y < w$ Then the possible values for y are 501, for a total of 167 possibilities.
11. **C** Since each row has a product of 2021, the product $(-1)^{4+6}$ has a value of 1 . But the product $(-1)^{4+6} = 2021$, and the product $4 \cdot 6 = 47$ (since $(-1)^{4 \cdot 6} = 1$).

15. C For 4 heads to appear before 2 tails, there can be 4 heads in a row or 4 heads out of 5 tosses. $P(4 \text{ heads}) = \frac{5^8}{6^5}$

20. **C** Method 1: Triangle ADP has area $\frac{1}{2}(3)(2) = 3$ and hypotenuse equal to $\sqrt{13}$. Since $\triangle BAQ \sim \triangle ADP$, $\triangle BQA \sim \triangle APD$. Then, $\triangle AQP \sim \triangle APD$ with scale factor $\frac{6}{\sqrt{13}}$. Thus, $\angle AQP$ is a right angle.

Then $\angle PCQ = \angle BQD = 90^\circ$.

23. C Let $U = \frac{t}{t^2 + 1}$ and $y = \frac{t}{t^2 + 1}$

Using the Law of Cosines on $\triangle ABD$,

$$AB^2 = AD^2 + BD^2 - 2 \cdot AD \cdot BD \cdot \cos \angle ADB$$

Finally, constructing radii AP and BP and noting that $\angle APD = 120^\circ$, use the Law of Cosines on $\triangle APD$.

$$AP^2 = AD^2 + PD^2 - 2 \cdot AD \cdot PD \cdot \cos 120^\circ$$

Therefore, the area of circle P is $\frac{1}{2} \pi r^2$

Method 2: Construct AP and