

1. The graph of the function  $\frac{3^{x-y}}{2^{x-y}}$  passes through only three of the four quadrants. Prove that the function is linear and identify, with proof, the quadrant through which the graph does not pass.

2. A

3. The coefficients  $a$ ,  $b$ , and  $c$  of the equation  $ax^2 + bx + c = 0$  are odd integers. Prove that there exists no ordered triple  $(a, b, c)$  for which the roots of the equation are rational.

4. A set of three or more distinct prime numbers is called *amazing* if the sum of every three of them is also a prime number. For example, the set  $\{11, 23, 37, 79\}$  is an amazing set of primes since  $11 + 23 + 37 = 71$  is prime,  $11 + 23 + 79 = 113$  is prime,  $11 + 37 + 79 = 127$  is prime, and  $23 + 37 + 79 = 139$  is prime. However, the set  $\{5, 7, 11, 13\}$  is not amazing since  $5 + 7 + 13 = 25$ .

a)



4. a) Any integer  $n$  can be written in the form  $n = 3k + r$  where  $k$  is an integer and  $r = 0, 1, \text{ or } 2$ . Let us refer to these as type  $r$ , where  $r = 0, 1, \text{ or } 2$ . The only prime number for which  $r = 0$  is 3 itself. Suppose  $S$  is an amazing set of four primes, one of which is 3. Represent the three remaining primes as  $\quad$ ,  $\quad$ , and  $\quad$ .

Claim: These three primes cannot all be of the same type  $r$ . Suppose they were. Then