



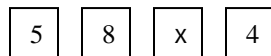
THE 2011-2012 KENNEBEC VALLEY REGIONAL HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

Each of the following 25 questions is worth 4 points. The total score is out of 100 points. Do not use a calculator. Each question has 5 possible answers. Only one answer is correct. You may use a pencil.

NO CALCULATORS

90 MINUTES

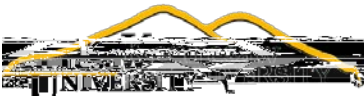


- In the puzzle at the right, the number in each empty square is obtained by adding the two numbers in the row directly above. For example,  $5 + 8 = 13$ . What is the value of  $x$ ?  
 (A) 2      (B) 3      (C) 6      (D) 7      (E) 9
- A circle passes through the points  $(0, 0)$ ,  $(0, 2)$  and  $(4, 0)$ . What is the area of this circle?  
 (A) 5      (B) 8      (C) 9      (D) 10      (E) 16
- Tom found the value of  $3^{21} = 10,4A0,353,20B$ . He found all the digits correctly except the fourth and last digits, denoted by A and B, respectively. What is the value of A?  
 (A) 0      (B) 2      (C) 3      (D) 6      (E) 8
- In determining standings in a certain hockey league, a team receives 3 points for each win, 1 point for each tie, and  $-1$  point for each loss. After playing 50 games, the Ducks have a total of 76 points. How many more wins than losses do the Ducks have at this time in the season?  
 (A) 12      (B) 13      (C) 14      (D) 15      (E) 16
- Let  $x = m + n$  where  $m$  and  $n$  are positive integers satisfying  $2^m + 2^n = 2^7$ . The  
 (A) 11      (B) 12      (C) 13      (D) 14      (E) 15

7. If the measure of  $\angle ABE$  is 6 degrees greater than the measure of  $\angle DCE$ , compute the number of degrees in the measure of  $\angle FD$ .
- (A) 6            (B) 8            (C) 10            (D) 12            (E) 16
8. If  $q$  and  $r$  are the zeros of the quadratic polynomial  $x^2 + 15x + 31$ , find the quadratic polynomial whose zeros are  $q + 1$  and  $r + 1$ .
- (A)  $x^2 + 17x + 31$     (B)  $x^2 + 15x + 33$     (C)  $x^2 + 13x + 17$   
(D)  $x^2 + 19x + 37$     (E) None of these
9. The makers of Delight Ice Cream put a coupon for a free ice cream bar in every 80<sup>th</sup> bar they make. They put a coupon for 2 free bars in every 180<sup>th</sup> bar and a coupon for 3 free bars in every 300<sup>th</sup> bar. If they put all three coupons in every  $n^{\text{th}}$  bar, compute  $n$ .
- (A) 1200    (B) 1800    (C) 2400    (D) 3600    (E) 5400
10. Starting at opposite ends of a straight moving walkway at an airport, which travels at a constant rate of  $k$  ft/sec, Don and Debbie walk towards each other (Don moving in the direction the walkway is moving, Debbie moving against the direction the walkway is moving). They meet at a point one-seventh of the way from one end of the walkway. If they were on a normal (non-moving) floor they would each walk at a rate of 3 feet per second. Determine the value of  $k$ .
- (A)



20. The number 2011 can be written as  $a^2 - b^2$  where  $a$  and  $b$  are integers. Compute the value of  $a^2 + b^2$ .
- (A) 2018041    (B) 2022061    (C) 2024072    (D) 2026085    (E) 2033051
21. Consider the following system of equations: (1)  $ax + by = c$  and (2)  $dx + ey = f$  ( $c \neq 0, f \neq 0$ ). When  $x = 0$ , equation (1) yields  $y = 3$  and (2) yields  $y = 6$ . When  $y = 0$ , (1) yields  $x = -3$  and (2) yields  $x = 3$ . What is the common solution  $(x, y)$  for the system?
- (A) (1, 2)    (B) (2, 6)    (C) (4, 1)    (D) (6, 2)    (E) (1, 4)
22. Rectangle ABCD has sides of length 3 and 4. Rectangle PCQD is similar to rectangle ABCD, with P inside rectangle ABCD. Compute the distance from P to AB.
- (A) 1    (B)  $\frac{4}{3}$     (C)  $\frac{7}{5}$     (D)  $\frac{21}{17}$     (E)  $\frac{27}{25}$
23. Let  $S(n) = [\sqrt{1}] + [\sqrt{2}] + \dots + [\sqrt{n}]$  where  $[k]$  is the greatest integer less than or equal to  $k$ . Compute the largest value of  $k < 2011$  such that  $S(2011) - S(k)$  is

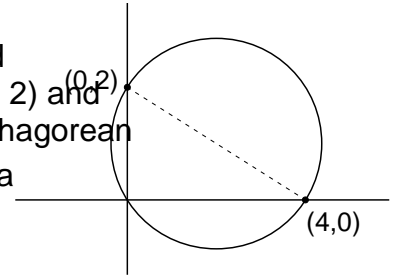


THE 2011–2012 KENNESAW STATE UNIVERSITY  
HIGH SCHOOL MATHEMATICS COMPETITION

SOLUTIONS

1. **A** The entries in the four empty boxes, from top to bottom and left to right  $x + 8$ ,  $x + 4$ ,  $x + 21$  and  $x + 12$ . Then  $(x + 21) + (x + 12) = 39$  or  $2x + 33 = 39$ , and  $x = 3$ .

2. **A** Because the angle at  $(0, 0)$  is a right angle, it is inscribed in a semicircle, which makes the segment connecting  $(0, 2)$  and  $(4, 0)$  a diameter. Using the distance formula (or the Pythagorean Theorem), the diameter of the circle is  $\sqrt{20}$ , making the area  $\frac{1}{4}(\sqrt{20})^2 = 5$ .



3. **D** Of course, one could compute the value of  $3^{21}$  directly, but that would take some time and might lead to careless errors. A more general approach is as follows. Let's find  $B$  first. The powers of 3, taken in order from  $3^1$  end in the repeating pattern 3, 9, 7, 1. Since 21 is one more than a multiple of 4,  $B = 3$ . Since  $3^{21}$  is a multiple of 9, its digits must sum to a multiple of 9. Since the known digits and  $B$  have a sum of 21, the missing digit  $A$  must be 6.
4. **B** Let  $W$  = the number of wins,  $T$  = the number of ties, and  $L$  = the number of losses.  $W + T + L = 50$  and  $3W + T = 100$ .

7. **A** Represent  $m'FBC$  as  $180 - (x + 6) = 174 - x$ . Also,  
 $m'DCE = m'BCF = x$ . Then  $m'AFD = 180 - (174 - x) - x = 6$ .

8. **C** We could find the zeros of the given polynomial, increase each by 1, and use them to find the answer. However, that is time consuming. Here are two shorter methods.

Method 1: IC BT /TT0 1 C BT /TT0 1 T4</MCID 6 >>BDC 0 -1.15 TD (8)Tj (.)L0k0 scn Tf

13. A  $\log_2 4 = 2$  and  $\log_2 2 = \frac{1}{2}$ . Using the triangle inequality, we have

$$\log_3 x < 2 + \frac{1}{2} \quad \text{and} \quad \log_3 x + \frac{1}{2} > 2$$

Therefore,  $\log_3 x < \frac{5}{2}$   $\checkmark$   $x < 3^{\frac{5}{2}}$  or  $x < 9\sqrt{3}$  and

$$\log_3 x + \frac{1}{2} > 2 \quad \checkmark \quad \log_3 x > \frac{3}{2} \quad \text{or} \quad x > 3\sqrt{3}.$$

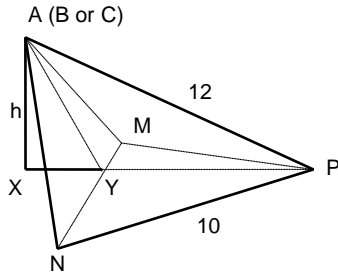
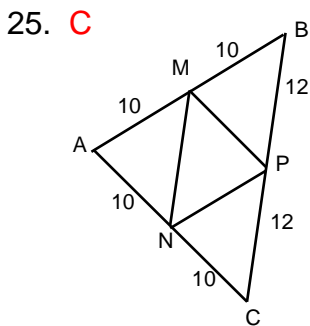
Therefore, the set of all possible values of  $x$  is  $3\sqrt{3} < x < 9\sqrt{3}$ . Since  $3\sqrt{3} \approx 5.2$  and  $9\sqrt{3} \approx 15.6$ , choice A (5) is the only choice that is not possible.

18. **E** Using  $x = 5$ , we obtain  $2f(5) + f(-4) = 25$   
Using  $x = -4$ , we obtain  $2f(-4) + f(5) = 16$ .  
Multiplying the first equation by 2 and subtracting the equations we obtain  
 $-3f(5) = -34$  from which  $f(5) = \frac{34}{3}$ .
19. **E** Let  $m$  be the slope of the line tangent to the ellipse. The equation of the tangent

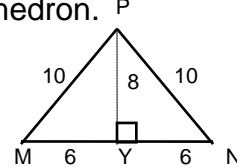


23. **D** Since  $\sqrt{1936} = 44$  and  $\sqrt{2025} = 45$ , all numbers from  $\lceil\sqrt{1936}\rceil$  to  $\lfloor\sqrt{2011}\rfloor$  must equal 44. If  $K \leq 1936$ ,  $S(2011) - S(K) = \lfloor\sqrt{2011}\rfloor + \lfloor\sqrt{2010}\rfloor + \dots + \lfloor\sqrt{K+1}\rfloor = 44(2011 - K) = (4)(11)(2011 - K)$ . Therefore  $S(2011) - S(K)$  will be a perfect square for  $2011 - K = 11$ , and  $K = 2000$ .

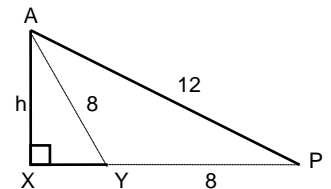
24. **C** & D  
 Before the operation, the sum of the squares of these two is  $2a^2 + 2a + 1 + a^2 = 3a^2 + 2a + 1$ , whereas after the operation the sum of the squares is simply  $2a^2$ . Since the other numbers do not change, we see that the sum of the squares of all nine numbers goes down by two during every operation. Because  $(6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) - (0^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2) = 182$ , Anne performed  $182 \div 2 = 91$  operations.



We need to find the area  $B$  of triangle  $MNP$  (whose sides are 10, 10, and 12) and the length of the altitude of the tetrahedron.



It is easy to see that the area of triangle  $MNP$  is  $\frac{1}{2}(8)(12)$  or  $B = 48$  square units.



Next we find the length of  $h$ . In the middle diagram above, triangle  $PYA$  has sides  $PA = 12$ ,  $PY = 8$  and  $AY = 8$ .

Thus, triangle  $PYA$  is obtuse, as shown. Using the Law of Cosines on triangle  $PYA$ .

$$144 = 64 + 64 - 128 \cos(\angle APY) \quad \text{and} \quad \cos(\angle AYP) = \frac{1}{8}$$

Therefore,  $\cos(\angle AYP) = \frac{1}{8}$ , making  $XY = 1$  and  $h = 3\sqrt{7}$ . Hence,  $V = \frac{1}{3}Bh = \frac{1}{3}(48)(3\sqrt{7}) = 48\sqrt{7}$

The desired ordered pair is  $(48, 7)$